

M252 L8 Practice Exam 5 sections 14.1-14.3, 14.5-14.7

1. Evaluate the following integral.

$$\int_9^{4y} \frac{6y}{x} dx$$

2. Evaluate the following integral.

$$\int_{4x}^{x^4} \frac{-10y}{x} dy$$

3. Evaluate the following integral.

$$\int_y^{\frac{\pi}{11}} \sin^3 3x \cos y dx$$

4. Evaluate the following iterated integral.

$$\int_2^6 \int_1^6 (4x + y) dy dx$$

5. Evaluate the following iterated integral.

$$\int_6^7 \int_1^{\sqrt{x}} 2ye^{-x} dy dx$$

6. Evaluate the following iterated integral.

$$\int_3^5 \int_y^{-2y} (-2 + 6x^2 + 6y^2) dx dy$$

7. Evaluate the following iterated integral.

$$\int_0^{\frac{\pi}{9}} \int_0^{11\cos\theta} r dr d\theta$$

8. Evaluate the following improper integral.

$$\int_2^{\infty} \int_0^{\frac{7}{x}} y dy dx$$

9. Use an iterated integral to find the area of the region bounded by

$$\sqrt{x} + \sqrt{y} = 2, \quad x = 0, \quad y = 0.$$

10. Use an iterated integral to find the area of the region bounded by

$$4x - 3y = 0, \quad x + y = 2, \quad y = 0.$$

11. Sketch the region,  $R$ , of integration and then switch the order of integration for the following integral.

$$\int_0^4 \int_0^{\sqrt{16-x^2}} f(x, y) dy dx$$

12. Sketch the region,  $R$ , whose area is given by the iterated integral below. Then switch the order of integration and evaluate the integral.

$$\int_{-9}^9 \int_{-\sqrt{81-x^2}}^{\sqrt{81-x^2}} dy dx$$

13. Evaluate the iterated integral below. Note that it is necessary to switch the order of integration.

$$\int_0^5 \int_x^5 e^{-8y^2} dy dx$$

14. Sketch the region  $R$  and evaluate the iterated integral.

$$I = \int_0^6 \int_{\pi/2}^3 (x+y) dy dx$$

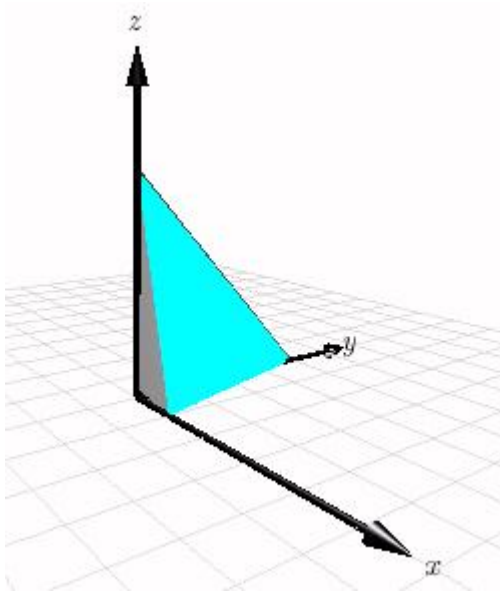
15. Set up an integral for both orders of integration, and use the more convenient order to evaluate the integral below over the region  $R$ .

$$\iint_R \frac{y}{x^2 + y^2} dA$$

$R$ : triangle bounded by  $y = 5x$ ,  $y = 8x$ , and  $x = 5$

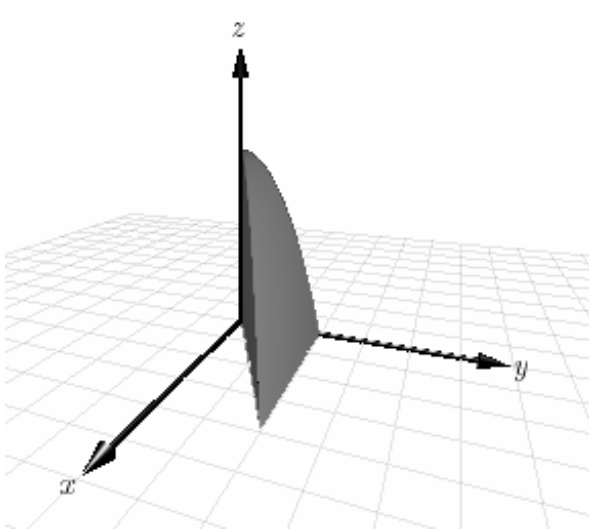
16. OMIT

17. Use a double integral to find the volume of the indicated solid.



$$4x + y + z = 2, \quad x > 0, y > 0, z > 0$$

18. Use a double integral to find the volume of the indicated solid.



$$z = 4 - y^2, \quad z > 0, x > 0, 3x < y < 2$$

19. Set up a double integral to find the volume of the solid bounded by the graphs of the equations given below.

$$z = xy, \quad z > 0, \quad x > 0, \quad 7x < y < 4$$

20. Set up a double integral to find the volume of the solid bounded by the graphs of the equations given below.

$$z = \frac{1}{25 + y^2}, \quad x = 0, \quad x = 2, \quad y \geq 0$$

21. Evaluate the iterated integral below. Note that it is necessary to switch the order of integration.

$$\int_0^5 \int_x^5 e^{-0.29y^2} dy dx$$

22. Find the average value of  $f(x,y)$  over the region  $R$  where:

$$\text{Average value} = \frac{1}{A} \iint_R f(x, y) dA$$

$$f(x, y) = xy$$

$R$ : rectangle with vertices  $(0,0)$ ,  $(1,0)$ ,  $(1,5)$ ,  $(0,5)$

23. Identify the region of integration for the following integral.

$$\int_0^{\pi/2} \int_0^{1+\sin\theta} r \theta dr d\theta$$

24. Evaluate the double integral below.

$$\int_0^{2\pi} \int_0^6 5 r^5 \sin \theta dr d\theta$$

25. Evaluate the double integral below.

$$\int_0^{\pi/7} \int_0^3 r^3 \sin \theta \cos \theta dr d\theta$$

26. Evaluate the double integral below.

$$\int_0^{\pi/2} \int_0^{5+3\sin\theta} \theta r dr d\theta$$

27. Evaluate the following iterated integral by converting to polar coordinates.

$$\int_0^5 \int_0^{\sqrt{25-x^2}} y \, dy \, dx$$

28. O M I T

29. O M I T

30. Use a double integral in polar coordinates to find the volume of the solid in the first octant bounded by the graphs of the equations given below.

$$z = x^4 y, \quad x^2 + y^2 = 16$$

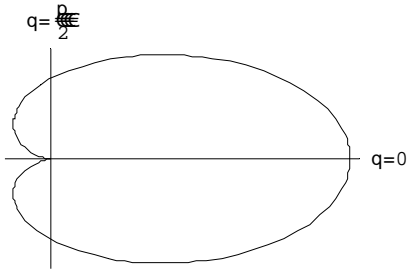
31. Use a double integral in polar coordinates to find the volume of the solid inside the hemisphere

$$z = \sqrt{81 - x^2 - y^2}$$

but outside the cylinder

$$x^2 + y^2 = 49.$$

32. Use a double integral to find the area enclosed by the graph of  $r = 5 + 5 \cos \theta$ .



33. O M I T

34. Find the area of the surface given by  $z=f(x,y)$  over the region  $R$ .

$$f(x, y) = -6 - 4x + 2y$$

$R$ : square with vertices  $(0,0)$ ,  $(4,0)$ ,  $(4,4)$ ,  $(0,4)$

35. Find the area of the surface given by  $z = f(x,y)$  over the region  $R$ .

$$f(x, y) = 10 + 2x - 4y$$

$R$ : rectangle with vertices  $(0,0)$ ,  $(4, 0)$ ,  $(4, 5)$ ,  $(0, 5)$

36. Find the area of the surface given by  $z = f(x,y)$  over the region  $R$ .

$$f(x, y) = -1 - x^2$$

$R$ : rectangle with vertices  $(0,0)$ ,  $(7, 0)$ ,  $(7, 7)$ ,  $(0, 7)$

37. Find the area of the surface given by  $z = f(x,y)$  over the region  $R$ .

$$f(x, y) = xy$$

$R$ :  $\{(x, y) : x^2 + y^2 \leq 121\}$

38. Find the area of the surface of the portion of the plane

$$z = 5 - 2x - 8y$$

in the first octant.

39. Set up a double integral that gives the area of the surface of the graph of  $f$  over the region  $R$ .

$$f(x, y) = x^4 - 3xy + 7y^3$$

$$R = \{(x, y) : -5 \leq x \leq 5, -2 \leq y \leq 2\}$$

40. Set up a double integral that gives the area of the surface of the graph of  $f$  over the region  $R$ .

$$f(x, y) = e^{7xy}$$

$$R = \{(x, y) : 0 \leq x \leq 2, 0 \leq y \leq 4\}$$

41. Evaluate the following iterated integral.

$$\int_0^6 \int_0^4 \int_0^5 (5x + 3y + z) \, dx \, dy \, dz$$

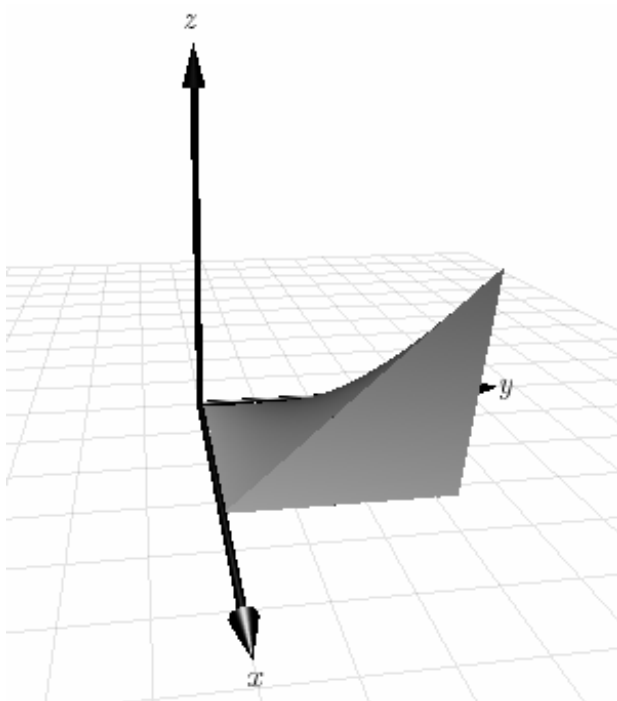
42. Evaluate the following iterated integral.

$$\int_1^8 \int_0^1 \int_0^x 2ze^{-x^2} \, dy \, dx \, dz$$

43. Set up a triple integral for the volume of the solid bounded by the coordinate planes and the plane given below.

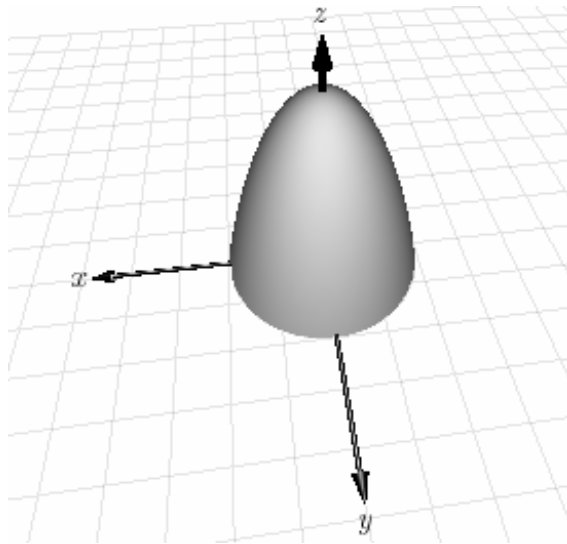
$$z = 6 - 2x - 4y$$

44. Use a triple integral for the volume of the solid shown below.



$$z = 12xy, \quad 0 \leq x \leq 5, \quad 0 \leq y \leq 6$$

45. Use a triple integral for the volume of the solid shown below.



$$z = 64 - x^2 - y^2, \quad z \geq 0$$

46. O M I T

47. Find the average value of the function  $f$  over the region in the first octant bounded by the coordinate planes, and the planes  $x = 4$ ,  $y = 3$ , and  $z = 2$ .

$$f(x, y, z) = x^3 + y^2 + z^4$$

48. Evaluate the following iterated integral.

$$\int_0^{\pi/2} \int_0^{1 \cos^4 \theta} \int_0^{1-r^2} r \sin \theta \, dz \, dr \, d\theta$$

49. Evaluate the following iterated integral.

$$\int_0^{\pi/13} \int_0^{\pi/13} \int_0^{\cos \theta} \rho^2 \sin \phi \cos \phi \, d\rho \, d\theta \, d\phi$$

50. Evaluate the following iterated integral.

$$\int_0^{2\pi} \int_0^{\pi} \int_3^5 \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

51. Convert the integral below from rectangular coordinates to cylindrical coordinates.

$$\int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{x^2+y^2}^9 x \, dz \, dy \, dx$$

52. Convert the integral below from rectangular coordinates to spherical coordinates.

$$\int_0^5 \int_0^{\sqrt{25-x^2}} \int_0^{\sqrt{25-x^2-y^2}} \sqrt{x^2 + y^2 + z^2} \, dz \, dy \, dx$$

53. Use cylindrical coordinates to find the volume inside the sphere

$$x^2 + y^2 + z^2 = 4$$

and above the upper nappe of the cone  $z^2 = (x^2 + y^2)$ .